OCR
RECOGNISING ACHIEVEMENT

ADVANCED SUBSIDIARY GCE

MATHEMATICS

Core Mathematics 1

4721

Candidates answer on the Answer Booklet

OCR Supplied Materials:
- 8 page Answer Booklet
- List of Formulae (MF1)

Other Materials Required:
None

Friday 9 January 2009
Morning

Duration: 1 hour 30 minutes

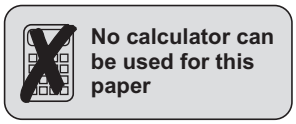
4721

INSTRUCTIONS TO CANDIDATES

- Write your name clearly in capital letters, your Centre Number and Candidate Number in the spaces provided on the Answer Booklet.
- Use black ink. Pencil may be used for graphs and diagrams only.
- Read each question carefully and make sure that you know what you have to do before starting your answer.
- Answer **all** the questions.
- Do **not** write in the bar codes.
- Give non-exact numerical answers correct to 3 significant figures unless a different degree of accuracy is specified in the question or is clearly appropriate.
- **You are not permitted to use a calculator in this paper.**

INFORMATION FOR CANDIDATES

- The number of marks is given in brackets **[ ]** at the end of each question or part question.
- **You are reminded of the need for clear presentation in your answers.**
- The total number of marks for this paper is **72**.
- This document consists of **4** pages. Any blank pages are indicated.



1 Express $\sqrt{45} + \dfrac{20}{\sqrt{5}}$ in the form $k\sqrt{5}$, where k is an integer. **[3]**

2 Simplify

(i) $\left(\sqrt[3]{x}\right)^6$, **[1]**

(ii) $\dfrac{3y^4 \times (10y)^3}{2y^5}$. **[3]**

3 Solve the equation $3x^{\frac{2}{3}} + x^{\frac{1}{3}} - 2 = 0$. **[5]**

4 **(i)** Sketch the curve $y = \dfrac{1}{x^2}$. **[2]**

(ii) The curve $y = \dfrac{1}{x^2}$ is translated by 3 units in the negative x-direction. State the equation of the curve after it has been translated. **[2]**

(iii) The curve $y = \dfrac{1}{x^2}$ is stretched parallel to the y-axis with scale factor 4 and, as a result, the point $P\,(1, 1)$ is transformed to the point Q. State the coordinates of Q. **[2]**

5 Find $\dfrac{\mathrm{d}y}{\mathrm{d}x}$ in each of the following cases:

(i) $y = 10x^{-5}$, **[2]**

(ii) $y = \sqrt[4]{x}$, **[3]**

(iii) $y = x(x + 3)(1 - 5x)$. **[4]**

6 **(i)** Express $5x^2 + 20x - 8$ in the form $p(x + q)^2 + r$. **[4]**

(ii) State the equation of the line of symmetry of the curve $y = 5x^2 + 20x - 8$. **[1]**

(iii) Calculate the discriminant of $5x^2 + 20x - 8$. **[2]**

(iv) State the number of real roots of the equation $5x^2 + 20x - 8 = 0$. **[1]**

7 The line with equation $3x + 4y - 10 = 0$ passes through point $A\,(2,\ 1)$ and point $B\,(10,\ k)$.

(i) Find the value of k. **[2]**

(ii) Calculate the length of AB. **[2]**

A circle has equation $(x - 6)^2 + (y + 2)^2 = 25$.

(iii) Write down the coordinates of the centre and the radius of the circle. **[2]**

(iv) Verify that AB is a diameter of the circle. **[2]**

8 **(i)** Solve the equation $5 - 8x - x^2 = 0$, giving your answers in simplified surd form. **[3]**

(ii) Solve the inequality $5 - 8x - x^2 \leqslant 0$. **[2]**

(iii) Sketch the curve $y = (5 - 8x - x^2)(x + 4)$, giving the coordinates of the points where the curve crosses the coordinate axes. **[5]**

9 The curve $y = x^3 + px^2 + 2$ has a stationary point when $x = 4$. Find the value of the constant p and determine whether the stationary point is a maximum or minimum point. **[7]**

10 A curve has equation $y = x^2 + x$.

(i) Find the gradient of the curve at the point for which $x = 2$. **[2]**

(ii) Find the equation of the normal to the curve at the point for which $x = 2$, giving your answer in the form $ax + by + c = 0$, where a, b and c are integers. **[4]**

(iii) Find the values of k for which the line $y = kx - 4$ is a tangent to the curve. **[6]**

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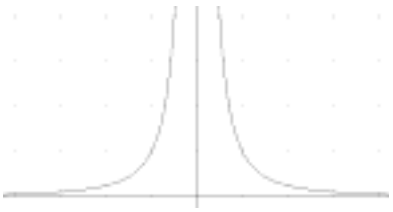
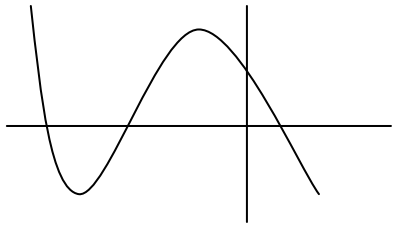
1	$3\sqrt{5}+\frac{20\sqrt{5}}{5}$ $=7\sqrt{5}$	B1 M1 A1 3 3	$3\sqrt{5}$ soi Attempt to rationalise $\frac{20}{\sqrt{5}}$ cao
2 (i)	x^2	B1 1	cao
(ii)	$\frac{3y^4\times1000y^3}{2y^5}$ $=1500y^2$	B1 B1 B1 3 4	$1000y^3$ soi 1500 y^2
3	*Let* $y=x^{\frac{1}{3}}$ $3y^2+y-2=0$ $(3y-2)(y+1)=0$ $y=\frac{2}{3},y=-1$ $x=\left(\frac{2}{3}\right)^3,x=(-1)^3$ $x=\frac{8}{27},x=-1$	*M1 DM1 A1 DM1 A1 ft 5 5	Attempt a substitution to obtain a quadratic or factorise with $\sqrt[3]{x}$ in each bracket Correct method to find roots Both values correct Attempt cube of at least one value Both answers correctly followed through **SR** If M1* not awarded, **B1** x = -1 from T & I
4 (i)		B1 B1 2	Excellent curve in one quadrant or roughly correct curves in correct 2 quadrants Completely correct
(ii)	$y=\frac{1}{(x+3)^2}$	M1 A1 2	$\frac{1}{(x\pm3)^2}$ $y=\frac{1}{(x+3)^2}$
(iii)	*(1, 4)*	B1 B1 2 6	Correct x coordinate Correct y coordinate

5 (i)	$\frac{dy}{dx} = -50x^{-6}$	M1 A1 2	kx^{-6} Fully correct answer
(ii)	$y = x^{\frac{1}{4}}$ $\frac{dy}{dx} = \frac{1}{4}x^{-\frac{3}{4}}$	B1 B1 B1 3	$\sqrt[4]{x} = x^{\frac{1}{4}}$ soi $\frac{1}{4}x^c$ $kx^{-\frac{3}{4}}$
(iii)	$y = (x^2 + 3x)(1 - 5x)$ $= 3x - 14x^2 - 5x^3$ $\frac{dy}{dx} = 3 - 28x - 15x^2$	M1 A1 M1 A1 4 **9**	Attempt to multiply out fully Correct expression (may have 4 terms) Two terms correctly differentiated from their expanded expression Completely correct (3 terms)
6(i)	$5(x^2 + 4x) - 8$ $= 5\left[(x+2)^2 - 4\right] - 8$ $= 5(x+2)^2 - 20 - 8$ $= 5(x+2)^2 - 28$	B1 B1 M1 A1 4	$p = 5$ $(x+2)^2$ seen or $q = 2$ $-8 - 5q^2$ or $-\frac{8}{5} - q^2$ $r = -28$
(ii)	$x = -2$	B1 ft 1	
(iii)	$20^2 - 4 \times 5 \times -8$ $= 560$	M1 A1 2	Uses $b^2 - 4ac$ 560
(iv)	2 real roots	B1 1 **8**	2 real roots
7(i)	$30 + 4k - 10 = 0$ $\therefore k = -5$	M1 A1 2	Attempt to substitute x = 10 into equation of line
(ii)	$\sqrt{(10-2)^2 + (-5-1)^2}$ $= \sqrt{64 + 36}$ $= 10$	M1 A1 2	Correct method to find line length using Pythagoras' theorem cao, dependent on correct value of k in (i)
(iii)	Centre (6, -2) Radius 5	B1 B1 2	
(iv)	Midpoint of AB = (6, -2) Length of AB = 2 x radius Both A and B lie on circumference Centre lies on line 3x + 4y – 10 = 0	B1 B1 2 **8**	One correct statement of verification Complete verification

8 (i)	$x = \dfrac{8 \pm \sqrt{(-8)^2 - (4 \times -1 \times 5)}}{-2}$	M1	Correct method to solve quadratic
	$= \dfrac{8 \pm \sqrt{84}}{-2}$	A1	$x = \dfrac{8 \pm \sqrt{84}}{-2}$
	$= -4 - \sqrt{21}$ or $= -4 + \sqrt{21}$	A1 3	Both roots correct and simplified
(ii)	$x \leq -4 - \sqrt{21}$, $x \geq -4 + \sqrt{21}$	M1	Identifying $x \leq$ their lower root, $x \geq$ their higher root
		A1 2	$x \leq -4 - \sqrt{21}$, $x \geq -4 + \sqrt{21}$ (not wrapped, no 'and')
(iii)		B1	Roughly correct negative cubic with max and min
		B1	(-4, 0)
		B1	(0, 20)
		B1	Cubic with 3 distinct real roots
		B1 5	Completely correct graph
		10	
9	$\dfrac{dy}{dx} = 3x^2 + 2px$	M1	Attempt to differentiate
		A1	Correct expression cao
	When $x = 4$, $\dfrac{dy}{dx} = 0$	M1	Setting their $\dfrac{dy}{dx} = 0$
	$\therefore 3 \times 4^2 + 8p = 0$	M1	Substitution of $x = 4$ into their $\frac{dy}{dx} = 0$ to evaluate p
	$8p = -48$		
	$p = -6$	A1	
	$\dfrac{d^2y}{dx^2} = 6x - 12$	M1	Looks at sign of $\dfrac{d^2y}{dx^2}$, derived correctly from their $\dfrac{dy}{dx}$, or other correct method
	When $x = 4$, $6x - 12 > 0$		
	Minimum point	A1 7	Minimum point CWO
		7	

10(i)	$\frac{dy}{dx} = 2x + 1$ $= 5$	M1 A1 2	Attempt to differentiate y cao
(ii)	Gradient of normal = $-\frac{1}{5}$ When $x = 2$, $y = 6$ $y - 6 = -\frac{1}{5}(x - 2)$ $x + 5y - 32 = 0$	B1 ft B1 M1 A1 4	ft from a non-zero numerical value in (i) May be embedded in equation of line Equation of line, any non-zero gradient, their y coordinate Correct equation in correct form
(iii)	$x^2 + x = kx - 4$ $x^2 + (1 - k)x + 4 = 0$ One solution => $b^2 - 4ac = 0$ $(1 - k)^2 - 4 \times 1 \times 4 = 0$ $(1 - k)^2 = 16$ $1 - k = \pm 4$ k = -3 or 5	*M1 DM1 DM1 A1 DM1 A1 6	Equating $y_1 = y_2$ Statement that discriminant = 0 Attempt (involving k) to use a, b, c from their equation Correct equation (may be unsimplified) Correct method to find k, dep on 1st 3Ms Both values correct
		12	